DYNAMIC FORMATION OF A BLOCK OF ELASTIC MATERIAL

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Deformation of a block of elastic material under dynamic loading is considered within the framework of the planar dynamic problem of elasticity theory. The numerical solution is based on Godunov's [1] finite difference methods and modifications thereof [2-4] for calculation of discontinuous flows. The stress-deformed state of the block is studied for various loading systems modeling the action of a high energy striker of various configurations. With a special blunt striker configuration a wedge of destroyed material is formed ahead of the striker, i.e., the instrument as it were sharpens itself at every blow, while ahead of the wedge there is a region of tensile stress, which can lead to the appearance of cracks.

1. Formulation of the Problem and Method of Solution. The functions $u(x, y, t)$, $v(x, y, t)$, $\sigma_1(x, y, t)$, $\sigma_2(x, y, t)$, $\tau_{12}(x, y, t)$ are defined, which in the range $0 \leqslant x \leqslant 1$, $0 \leqslant y \leqslant 1$, $0 \leqslant t \leqslant T_*$ satisfy the system of equations (see [2-4])

$$\rho\frac{\partial u}{\partial t}-\frac{\partial \sigma_1}{\partial x}-\frac{\partial \tau_{12}}{\partial y}=0,\ \rho\frac{\partial v}{\partial t}-\frac{\partial \tau_{12}}{\partial x}-\frac{\partial \sigma_2}{\partial y}=0,$$
$$\frac{\partial \sigma_1}{\partial t}-(\lambda+2\mu)\frac{\partial u}{\partial x}-\lambda\frac{\partial v}{\partial y}=0,\ \frac{\partial \sigma_2}{\partial t}-\lambda\frac{\partial u}{\partial x}-(\lambda+2\mu)\frac{\partial v}{\partial y}=0, \quad (1)$$
$$\frac{\partial \tau_{12}}{\partial t}-\mu\frac{\partial v}{\partial x}-\mu\frac{\partial u}{\partial y}=0,$$

initial conditions

$$u=v=\sigma_1=\sigma_2=\tau_{12}=0 \quad (2)$$

and boundary conditions

$$\sigma_1=\tau_{12}=0\ (x=0,\ 1);\ u=v=0$$
$$(y=0);\ \tau_{12}=0,\ \sigma_2=-1\ (y=1). \quad (3)$$

Here u, v are velocities along the axes Ox, Oy; σ_1, σ_2 are normal stresses along those axes; τ_{12} is the tangent stress; ρ is density; λ, μ are Lamé parameters.

The problem of Eqs. (1)-(3) must now be solved numerically. We will use the algorithm of [4], which developed the method of Godunov and Ivanov [1, 2] for calculation of discontinuous solutions. The method of [1] has the property of monotonicity and is of first order accuracy. In each layer in time the solution is considered piecewise-continuous, and expressions for the decay of an arbitrary discontinuity are used for the solution for the next stage. In [2] a method was proposed for approximating the solution with piecewise-linear polynomials. This method corresponds well to problems characterized by the presence of previously unknown discontinuity surfaces [3]. In [4] a set of methods was developed allowing finding a solutions with greater accuracy then the techniques of [1, 2], while retaining their approach. The improvement was achieved by including a larger number of terms in auxiliary equations and finally producing a family of difference schemes containing a larger number of order parameters (dissipation constants).

The values of the eight control parameters of the problem of Eqs. (1)-(3) were chosen such that the slopes of the calculated fronts of the incident (Fig. 1, solid lines) and reflected (dashed lines) were maximal (arrows indicate the direction of front motion). Numerical experiments were performed for a model material with characteristics $\rho = 1$, $\lambda = 3/7$, $\mu = 2/7$, a block of unit dimensions being considered, while at the initial moment $t = 0$ on the boundary $y = 1$ a unit normal compressive load was specified [see Eqs. (1)-(3)]. Calculation was terminated after the period T_*, when the disturbance reflected from the rear boundary returned to the original one. These parameter values were used to solve all subsequent problems differing from that of Eqs. (1)-(3) only in boundary conditions.

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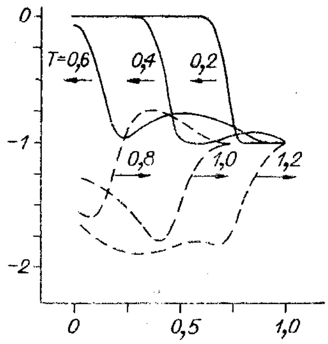

Fig. 1

Three different methods for loading the block of material were considered: a) a blunt instrument with slippage along the block surface; b) a blunt instrument with tangent stress present; c) a blunt hollow instrument with slippage along the block surface, namely:

$$
\begin{gathered}
u=\tau_{12}=0\ (x=0);\ \sigma_1=\tau_{12}=0\ (x=1);\ u=v=0 \\
(y=0);\ \tau_{12}=0\ (y=1); \\
\sigma_2=-1,\ 0\leqslant x\leqslant 0{,}225;\ \sigma_2=20x-5{,}5,\ 0{,}225<x\leqslant 0{,}275; \\
\sigma_2=0,\ 0{,}275<x\leqslant 1\ (y=1);
\end{gathered} \tag{4a}
$$

$$
\begin{gathered}
u=\tau_{12}=0\ (x=0);\ \sigma_1=\tau_{12}=0\ (x=1);\ u=v=0\ (y=0); \\
\tau_{12}=0{,}5\sin^2 4\pi x,\ 0\leqslant x\leqslant 0{,}25;\ \tau_{12}=0,\ 0{,}25<x\leqslant 1 \\
(y=1); \\
\sigma_2=-1,\ 0\leqslant x\leqslant 0{,}225;\ \sigma_2=20x-5{,}5,\ 0{,}225<x\leqslant 0{,}275; \\
\sigma_2=0,\ 0{,}275<x\leqslant 1\ (y=1);
\end{gathered} \tag{4b}
$$

$$
\begin{gathered}
u=\tau_{12}=0\ (x=0);\ \sigma_1=\tau_{12}=0\ (x=1);\ u=v=0 \\
(y=0);\ \tau_{12}=0\ (y=1); \\
\sigma_2=0,\ 0\leqslant x\leqslant 0{,}1;\ \sigma_2=-1,\ 0{,}1<x\leqslant 0{,}225;\ \sigma_2=20x-5{,}5, \\
0{,}225<x\leqslant 0{,}275;\ \sigma_2=0,\ 0{,}275<x\leqslant 1\ (y=1).
\end{gathered} \tag{4c}
$$

The problems thus formulated, those of Eqs. (1), (2), (4a); (1), (2), (4b); and (1), (2), (4c) model central loading, therefore half the block is considered, with u = 0 at x = 0. Boundary conditions (4a), (4b), (4c) differ only in the values of τ_{12}, σ_2 on the boundary y = 1. All boundary conditions for σ_2 model loading by a blunt instrument with curved outer edge.

2. Evaluation of Results. Calculation results for the five successive times $0.2T_*$; $0.4T_*$; $0.6T_*$; $0.8T_*$; T_* were depicted by stress isolines σ_1, σ_2, τ_{max}. Some of the results are shown in Figs. 2-5. Figure 2 shows σ_2 isolines for $t = 0.8T_*$ and boundary conditions (4a)-(4c), with curves corresponding to conditions (4a) being labeled a, those for Eq. (4b), b, and (4c), c. Figure 3 shows σ_1 isolines for $t = 0.6T_*$, and Figs. 4, 5, τ_{max} isolines for $t = 0.6T_*$ and $t = 0.8T_*$, respectively. Comparing the graphs we can note the following features of the dynamic block deformation process. The character of block loading, i.e., the various values of τ_{12}, σ_2 on the edge $y=1$, $0\leqslant x\leqslant a=0.25$, exert a significant effect on the distribution of all stresses σ_1, σ_2, τ_{max} over the entire depth of the block in a band the width of which is not greater than 0.8a. Beyond the limits of this band the effect of the character of loading is practically unnoticable (Figs. 2-5). Within the region where loading has an effect, the problem with boundary conditions (4b) is a "core" with center at

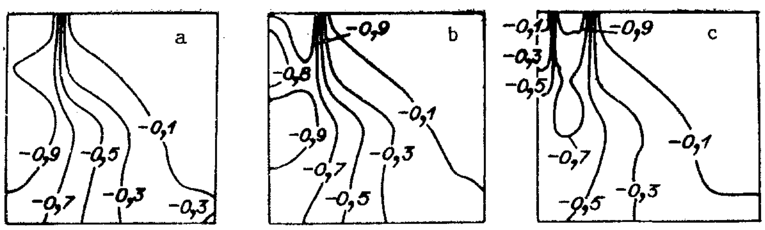

Fig. 2

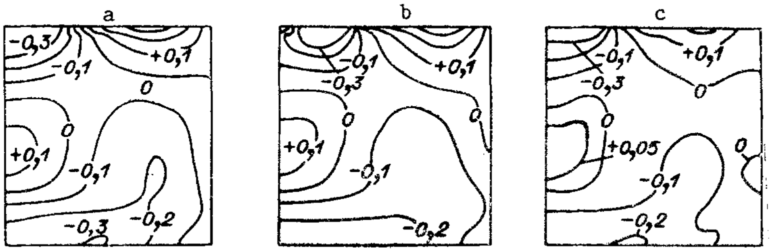

Fig. 3

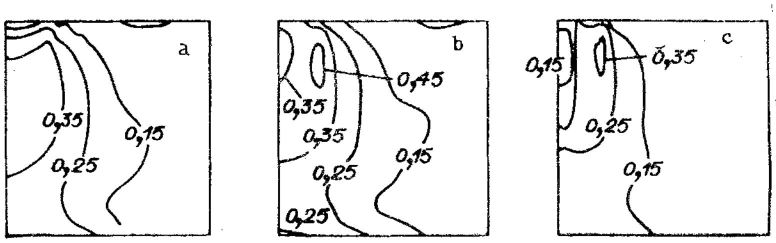

Fig. 4

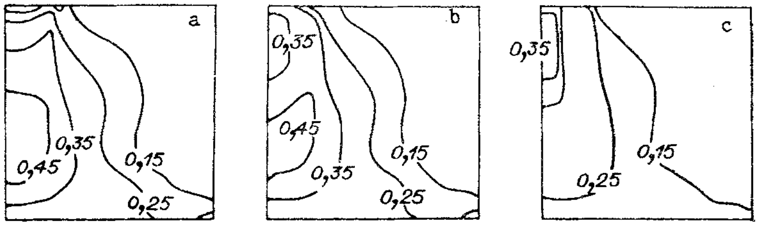

Fig. 5

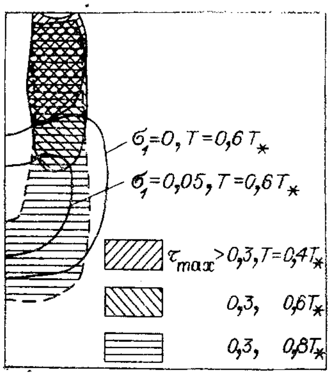

Fig. 6

the point (0, 0.8) of lowered stresses σ_2 and τ_{max} (Figs. 2b, 4b, 5b) while for Eq. (4c) we have a "wedge" of reduced stress σ_2 (Fig. 2c) and τ_{max} (Figs. 4c, 5c). In the vicinity of the point (0, 1) the problem with boundary conditions (4b) constantly maintains a tensile stress σ_1 (Fig. 3b) and the highest stress τ_{max} (Figs. 4b, 5b). The σ_1 stress distribution in the problems with boundary conditions (4a) and (4c) practically coincide over the entire region (Fig. 3a, 3c).

The isolines shown in Fig. 6 for τ_{max} at various times can be related to formation of a shear crack, if we use Zhurkov's kinetic concept of failure [5]. According to this concept the fundamental characteristic of failure is its duration (see, for example, [6]). For a given inensity of tensile or shear stress the duration is identified as the time from which the moment stress is applied until the body separates into places. After accumulation of damage in the block material a system of microcracks is formed in the region where the load is applied (cross-hatched region in Fig. 6); these cracks merge into a magistral, forming as it were a "sharp" instrument ahead of the striker with a central orifice, which permits breakage of the block into parts, as encouraged by the presence of zones of tensile stress ahead of the the "sharp instrument." Thus, another method of block failure aside from the traditional one is possible.

The above does not agree with the assertion that "with increase in blow energy the effect of instrument geometry on efficiency of rock breakup decreases significantly..." [7, p. 29]. The numerical experiments performed show that with increase in blow energy one can in principle achieve destruction of the block material with every blow.

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ANALYSIS OF CREEP IN A RECTANGULAR PLATE WITH A CIRCULAR ORIFICE UNDER TENSION

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A finite element solution of the problem is offered using the theory of strengthening type creep. Because of creep the stress concentration at the edge of the orifice is reduced, and displacement as a function of reduced time increases at an almost constant rate. Moiré type displacement isoline patterns are presented.

Fundamental Equations. Expressions for the deformations in terms of displacements, the equilibrium equation, and relationships between stresses and deformations in the plane stressed state are taken in the form [1, 2]

$$e_{11} = u_{1,1},\ e_{22} = u_{2,2},\ 2e_{12} = u_{1,2} + u_{2,1},$$
$$\sigma_{11,1} + \sigma_{12,2} = 0,\ \sigma_{12,1} + \sigma_{22,2} = 0,$$
$$e_{11} = E^{-1}(\sigma_{11} - \nu\sigma_{22}) + \rho_{11},\ e_{22} = E^{-1}(\sigma_{22} - \nu\sigma_{11}) + \rho_{22},$$
$$e_{12} = (1 + \nu)E^{-1}\sigma_{12} + \rho_{12}.$$

Here E is Young's modulus, ν is the Poisson coefficient, u_i are the displacements; e_{ij}, the deformations; σ_{ij}, the stresses; ρ_{ij} are the creep deformations (i, j = 1, 2) in the Cartesian coordinate system x_1, x_2; the subscripts 1 and 2 following the comma indicate partial differentiation with respect to x_1 and x_2, respectively.

In contrast to [3-7], to calculate plate creep we will use the strengthening type creep theory of [2, 8]. In the uniaxially stressed state the creep deformation ρ and the stress σ as a function of time t are interrelated in this theory by an expression

$$\rho^k d\rho/dt = a\sigma^n, \tag{1}$$

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